BEEF CATTLE FEED INTAKE AND GROWTH: APPLICATION OF AN EMPIRICAL BAYES DERIVATION OF THE KALMAN FILTER APPLIED TO A NONLINEAR DYNAMIC MODEL

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Story in Brief

The empirical Bayes derivation of the Kalman filter has been applied to a nonlinear dynamic, mechanistic model of beef cattle feed intake and growth. This recursive procedure updates predictions of cattle feed intake and gain by multiplying the model's estimates by appropriate factors based on the statistical weighting of previous and current observation of animal performance. When the empirical Bayes procedure was applied to 200 pens of feedlot cattle, predictions of daily feed intake and weight gain for the whole feeding period made after only two observations of intake and gain were 41 and 26% more precise than original estimates. Mean absolute error for predictions of daily intake and gain after two observations were .42 and .14 kg/d. These estimates were .84 and .18 kg/d more precise than model predictions without adjustment after two periods for intake and gain, respectively. This improvement in precision demonstrates how a general model can be adjusted based on initial observations to fit performance in a specific situation.

(Key Words: Bayes Statistics, Beef Cattle Growth, Dynamic Models)

Introduction

Models often combine a series of observed relationships to describe the general relationships among factors. Hence, the derived model is general and imprecise as applied to an individual case. Yet predicting future behavior is one of a model's most important functions. As opposed to more empirical regression models which are calculated from the observed data, dynamic models based on the underlying mechanism of a system can be used to estimate response beyond that used to derive the model (Oltjen, 1984). Even though regression provides a precise estimate within the range of data available to estimate the regression parameters, extrapolating beyond this range is very risky. Mechanistic models (those based on fundamental biological relationships), on the other hand, can be extrapolated with some confidence. Extrapolation accuracy depends simply on the accuracy of the model's simulation of the fundamental biology. Application of such complex models to predict future system behavior can be enhanced by adjusting the model based on information of prior behavior of similar systems and current observation of the system under consideration using empirical Bayes procedures (Allen and Jordan, 1982). The necessary tool is called the Kalman filter, a recursive procedure for the prediction of the state of a linear dynamic system (Kalman, 1960). Direct application requires adaptation to nonlinear systems. Although the Kalman filter has been applied to many problems in control engineering and physical science, its application to biological problems has been limited (Meinhold and

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Singpurwalla, 1983). It has been applied to a dynamic linear system of bovine lactation yield (Goodall and Sprevak, 1985). The researchers found the method gave accurate estimates of total milk yield after only a few observations during the very early stages of lactation. The objective of this research was to develop the theoretical basis for application of the empirical Bayes derivation of the Kalman filter to dynamic systems described by nonlinear sets of differential equations. The theory was tested by applying it to a nonlinear, dynamic system of animal growth and development.

Materials and Methods

Generally, the approach is similar to that for linear systems (Harrison and Stevens, 1976). In order to apply the linear theory, the variables from the dynamic model which are easily observed in the biological system are identified. For each of these variables, a linear function relating predicted and actual values is assumed. Parameters of this function are updated using the Bayesian Kalman filter based on the earlier observations. The updated function is used to modify the model to predict future behavior. To implement this procedure, initial values for the parameters and their variance-covariance matrix must be specified. If no previous information is available, they are set to represent no deviation between the predicted and observed values. Their variance is the identity matrix which allows the incoming observations to dictate convergence. However, where data is available or the procedure has been applied previously, the initial values are used to more properly weight the Bayes estimates.

To test this theory, dynamic models of bovine growth and composition (Oltjen, 1983) and feed intake (Plegge et al., 1984) were integrated. The growth model is based on the mechanistic relationships between increasing cell numbers and cell size, genetic potential, energy expenditure and environmental influences (eg. feed intake) of a growing and maturing animal. Nutrient intake is brought into the system with the daily feed intake model, which, in contrast to the growth model, is an empirical fit of a large data set which includes the effects of body weight, initial feeding period weight, feed metabolizeable energy and implant treatment. Average daily body weight gain and feed dry matter intake form the set of observable variables to be tested in the Bayes procedure. The linear function used was a simple multiplicative adjustment for both variables.

The test data set consisted of feed intake and body weight gain for 200 pens of typical feedlot steers fed in research trials at Oklahoma State University at period intervals (usually 28 days, Table 1). Initial values for the linear function's parameters were determined using preliminary analysis of data from 18 additional independent pens not used in the empirical Bayes analysis.

For evaluation, the model was run for the entire feeding period both without any adjustment (M) and using the empirical Bayes procedure (eB). Another multiplicative adjustment was tested which was based on the least-squares mean estimation procedure (LS) in which each period's performance was used to establish adjustments which were multiplied by model estimates to predict feed intake and gain. Thus, LS could not be estimated until after the first period's performance had been observed. The recursive eB and LS procedures were applied at the beginning of each period using the latest period's best estimate.

Table 1. Description of the steer data used for analysis.

Item	Number of pens	Mean	Standard deviation	Range
Days on feed	200	141.5	26.2	111 - 201
Periods (final day)				
1	200	36.7	11.9	27 - 57
2	200	74.6	23.9	55 - 121
3	176	101.3	21.1	83 - 140
4	130	121.1	11.2	111 - 147
5	50	140.0	.0	140 - 140
6	50	166.9	3.3	160 - 169
7	24	199.1	2.5	196 - 201
Weight (kg)				
Initial	200	290.0	41.6	193 - 357
Final	200	496.3	25.6	420 - 549
Daily gain	200	1.47	.13	1.18 - 1.79
Feed dry matter				
Intake (kg/d)	200	8.88	.85	7.07 - 10.80
ME (Mcal/kg)	200	3.08	.12	2.90 - 3.57

Two measures were evaluated. First predictions of daily feed intake and weight gain for each period for M, eB and LS were compared with the period's observed feed intake and weight gain. The second comparison was between observed and predicted feed intake and weight gain for the entire feeding period. The relationship among residual and absolute residual errors with time on feed were examined by regression analysis. Higher order terms for days on feed and analysis by period also were included in a sensitivity analysis of residual errors.

Results and Discussion

The unadjusted model (M) underpredicted both feed intake and daily gain for each period (Table 2). Similarly, total feeding period performance was better than the initial model predicted (Table 3). However, errors of prediction for daily feed intake and gain in each feeding period and for total time on feed demonstrate one advantage of using the empirical Bayes procedure, particularly for estimating performance at the end of the first few periods. The residual inaccuracy based on eB was less than the bias of M or LS for all feed intake and most gain predictions made for periods 2 through 5. Only estimates for period 2 and 5 daily gain and period 5 total feeding period gain were lower for M or LS procedures. In general, eB standard deviations also were smaller during the first few periods, particularly as related to the LS procedure. As fewer pens with 5 or more periods were available, these estimates are less reliable. The rapid decrease in bias and improvement in precision of the eB procedure by including only one observation of performance (period 2 estimate) demonstrates the immediate usefulness of the empirical Bayes procedure.

The least-squares procedure required several observations before accurate predictions were obtained (Figures 1 and 2). At the beginning of period 2, mean absolute errors for eB were .12 and .33 kg/d less than

Table 2. Residual (predicted-observed) daily feed intake and gain for individual feeding periods using model (M) and least-squares (LS) or empirical Bayes (eB) adjusted model estimates.

		Feed intake (kg/d)						Daily gain (kg/d)					
		M		LS		eB		M		LS		eB	
Period	Number of Pens	Mean	(SD)	Mean	(SD)	Mean	(SD)	Mean	(SD)	Mean	(SD)	Mean	(SD)
1	200	-1.40	(1.14)	--	--	-.63	(1.25)	-.48	(.37)	--	--	-.30	(.39)
2	200	-1.48	(1.23)	.76	(2.02)	-.09	(1.05)	-.19	(.31)	.88	(1.11)	.21	(.29)
3	176	-1.15	(1.03)	.56	(.76)	.42	(.69)	-.37	(.28)	.15	(.39)	.08	(.32)
4	130	-1.03	(.89)	.42	(1.03)	.35	(.93)	-.26	(.27)	.10	(.33)	.10	(.30)
5	50	-1.25	(.91)	.92	(.51)	.79	(.56)	-.24	(.22)	.09	(.26)	.18	(.27)
6	50	-.71	(1.00)	1.06	(.75)	.96	(.71)	.04	(.35)	.24	(.36)	.32	(.37)
7	24	.32	(.66)	1.54	(.78)	1.57	(.73)	.29	(.17)	.41	(.21)	.50	(.23)

Table 3. Residual (predicted-observed) daily feed intake and gain for the total feeding period using model (M) and least-squares (LS) or empirical Bayes (eB) adjusted model estimates.

		Feed intake (kg/d)						Daily gain (kg/d)					
		M		LS		eB		M		LS		eB	
Period	Number of Pens	Mean	(SD)	Mean	(SD)	Mean	(SD)	Mean	(SD)	Mean	(SD)	Mean	(SD)
1	200	-1.12	(.84)	--	--	-.23	(.82)	-.26	(.22)	--	--	-.05	(.22)
2	200	-1.12	(.84)	.31	(.89)	.16	(.66)	-.26	(.22)	.46	(.53)	.14	(.23)
3	176	-1.23	(.81)	.43	(.58)	.20	(.50)	-.31	(.18)	.18	(.27)	.09	(.17)
4	130	-1.10	(.79)	.25	(.43)	.19	(.40)	-.28	(.18)	.06	(.13)	.06	(.18)
5	50	-1.51	(.86)	.33	(.19)	.31	(.29)	-.25	(.11)	.04	(.09)	.11	(.08)
6	50	-1.51	(.86)	.11	(.17)	.16	(.23)	-.25	(.11)	-.01	(.07)	.26	(.18)
7	24	-1.13	(.94)	-.02	(.18)	.15	(.11)	-.23	(.12)	-.04	(.07)	.22	(.25)

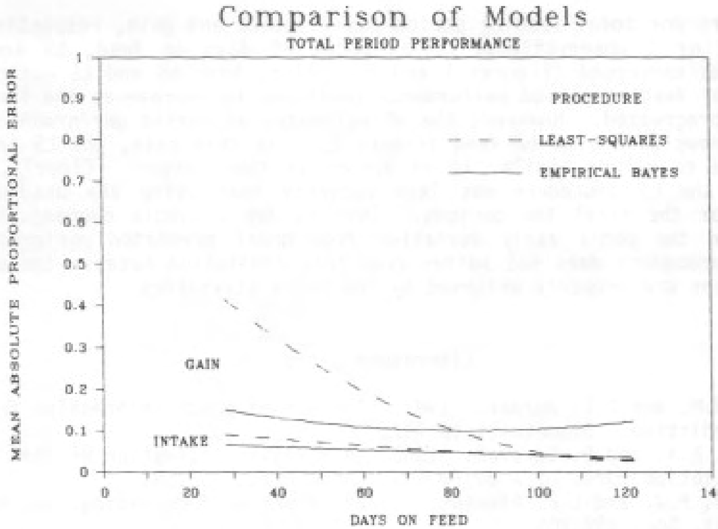

Figure 1. Comparison of least-squares and empirical Bayes procedure mean absolute proportional prediction error of total feeding period feed intake and body weight gain as days on feed increase.

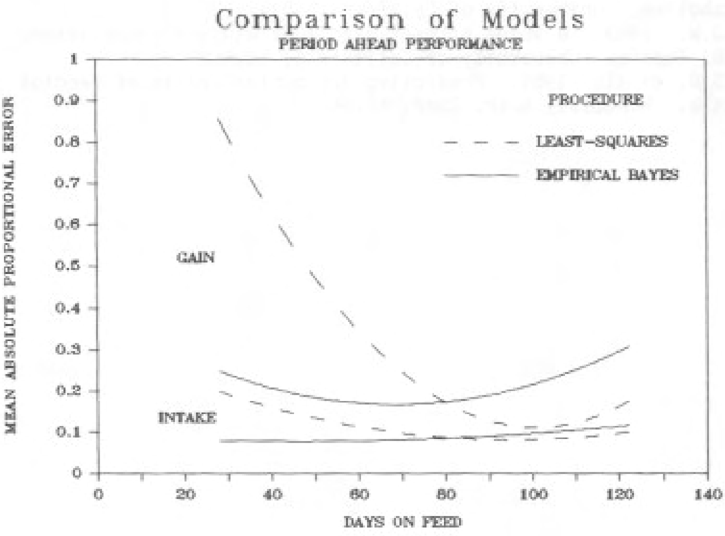

Figure 2. Comparison of least-squares and empirical Bayes procedure mean absolute proportional prediction error of period ahead feed intake and body weight gain as days on feed increase.

LS errors for total feeding period daily intake and gain, respectively. After 2 or 3 observations, or at about 80 days on feed, LS and eB estimates converged (Figures 1 and 2). Also, both eB and LS estimates for total feeding period performance continued to improve as the feeding period progressed. However, the eB estimates of period performance did not improve with time on feed (Figure 2). In this case, the LS errors declined to values similar to eB errors as time passed. Clearly, the use of the LS procedure was less accurate than using the unadjusted model for the first few periods. This is due a simple overadjustment based on the pen's early deviation from model predicted performance; the eB procedure does not suffer from this limitation because the early deviations are properly weighted by the Bayes statistics.

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