

USE OF A STOCHASTIC PRODUCTION FUNCTION TO EVALUATE THE EFFECT OF ENERGY SUPPLEMENTATION OF WHEAT PASTURE STOCKER CATTLE ON PRODUCTION RISK

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Story in Brief

Effects on production risk of feeding a moderate amount of two different types on energy supplements to growing cattle on wheat pasture were examined. A stochastic production function was estimated, which provides a means of assessing effects of production inputs on both expected (projected) weight gains and their variability. Results indicate that both expected weight gains and their variance depends on the level of energy supplementation and forage availability. Supplemental energy is shown to reduce the variability of weight gain, and thus is considered a risk-reducing input. The results also indicate that weight gain variability decreases as forage availability increases. By increasing the certainty of cattle weights off wheat pasture, break-even selling prices can be more accurately calculated. This, in turn, will greatly aid marketing and retained ownership decisions.

(Key Words: Wheat Pasture, Energy Supplementation, Production Risks.)

Introduction

Growing cattle to heavier weights on wheat pasture is a major beef cattle production program in the southern Great Plains. Weight gains by stocker cattle grazing wheat pasture are potentially excellent because of its high quality. However, gains are often less than expected because of inadequate amounts of available forage. It is often stated that feeding moderate amounts of an energy supplement to growing cattle on wheat pasture is a way of increasing stability of the enterprise, improving predictability of cattle performance and increasing stocking density during the fall-winter grazing period. Measurements with regard to improving

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predicability of cattle performance (or of decreasing variability of performance) are generally lacking. During the 1989-90, 1990-91 and 1991-92 wheat pasture years, studies were conducted to compare moderate amounts (i.e., .66% of body wt) of high-starch versus high-fiber energy supplements for stocker cattle grazing wheat pasture. Details of the studies have been reported by Horn et al. (1991, 1992 and 1995), and Cravey et al. (1993). In this paper, a stochastic production function is estimated to determine the effect of energy supplementation on both expected weight gains and the variability of gains (i.e., production risk) of growing cattle on wheat pasture.

Materials and Methods

Most production functions indicate the relationship between the expected value of the dependent variable (e.g., weight gain) and various levels of the independent variables. The basic deterministic production function may be expressed as:

$$G = f(X_1, \dots, X_n) + u_i \quad [1]$$

where G is weight gain, X_1, \dots, X_n is a vector of input levels, and u_i is the disturbance term. In this paper, a stochastic production function is used which not only provides an estimate of the effect of the independent variables on expected weight gain, but also their effect on the variability of weight gain. The variability of weight gain is considered a measure of production risk. Thus, if the variability of weight gain increases as the level of input is increased, the input is considered a "risk-increasing" input; conversely, if the variability of weight gain is decreased as input is increased, the input is considered "risk-decreasing".

In the basic regression model, it is assumed that the disturbance terms all have the same variance, σ_u^2 . This condition of constant variance is known as homoskedasticity. It may be the case, however, that all of the disturbance terms do not have constant variance. For example, the variance around the regression equation relating weight gain to the quantity of feed input may increase (decrease) as input level is increased. This condition is referred to as heteroskedasticity. In this case, the use of ordinary least squares is inappropriate, but maximum likelihood procedures yield unbiased and efficient estimators (Judge et al., 1988).

In this paper, a stochastic production function of the type proposed by Just and Pope (1979) is used to determine the effect of forage availability and energy supplement on the expected value and variability of weight gain of stocker cattle grazing wheat pasture. The generalized form of the production model includes two functions: one which specifies the effects of

inputs on the mean of output and another which specifies the effects of input on the variance of output. Such a function is given by:

$$G = f(X_1, \dots, X_n) + e_i \quad [2]$$

with

$$e_i = \varepsilon_i h^{1/2}(X_1, \dots, X_n) \quad [3]$$

where e_i is the error term and ε_i is assumed normally distributed with a mean of zero and variance equal to one. Equation [3] implies that the error term is heteroskedastic since its variance depends upon input levels. In this specification, the deterministic component is represented by $E(G) = f(X_1, \dots, X_n)$ and the stochastic component by $V(G) = h(X_1, \dots, X_n, \alpha)$, where $V(\cdot)$ denotes the variance operator.

Stocker cattle production characteristics fundamentally determine the functional form of the deterministic component (equation [2]). Marginal products must be positive over some range of the sample data; second derivatives should be negative. Each additional unit of supplement input may result in less additional weight gain than the previous one since, in general, nutrient requirements per pound of weight gain increase at heavier weights (Epplin et al., 1981). Given that time series and cross-sectional data are used, time effects are accounted for in the production function specification through the use of dummy variables. Plot effects are not necessary since the cross-sectional units were in very close proximity in the original experiment. Time effects are important since the different cross-sections were affected by the same weather conditions each year.

For the stochastic component (equation [3]), it is assumed that the logarithm of the variance of weight gains is a linear function of the exogenous variables energy supplement, initial calf weight, and pounds of available forage. This is referred to as multiplicative heteroskedasticity because different components of the variance are related multiplicatively (Judge et al., 1988). In addition, time effects are assumed fixed (which implies the use of year dummy variables).

The specified model is:

$$\ln GN = \beta_0 + \sum_{t=1}^{T-1} \delta_t D_t + \beta_1 \ln INWT + \beta_2 \ln PF + \beta_3 \ln EN \quad [4]$$

$$+ \beta_4 \ln(PF) \ln(EN) + e_i$$

with

$$e_i = \varepsilon_i h^{1/2}(D_t, INWT, EN, PF, \mathbf{a}), \quad [5]$$

where GN denotes daily rate of weight gain (kg/head/grazing day), INWT is the initial calf weight (kg/head), EN is daily quantity of energy supplement

fed (Mcal/grazing day), PF is the level of forage availability (kg/steer day), and the D_t 's are year dummy variables. ε_i is normally distributed with $E(\varepsilon_i) = 0$ and $E(\varepsilon_i^2) = 1$.

The error term, e_i , is normally distributed with mean zero and variance

$$h(D_p, INWT, EN, PF, \mathbf{a}) = \exp(\alpha_0 + \alpha_1 INWT + \alpha_2 EN + \alpha_3 PF + \sum_{t=1}^{T-1} \gamma_t D_t).$$

The model is estimated by using maximum likelihood procedures. Given the importance of the stochastic component in modeling production risk, it is useful to test the assumption that variance depends on the specified exogenous variables.

Test 1: Test for Multiplicative Heteroskedasticity

$$H_0: \alpha_1 = \alpha_2 = \alpha_3 = 0$$

$$H_1: \text{not all } \alpha_i \text{'s are zero } (i = 1, 2, 3)$$

Similarly, the significance of time effects on mean and variance of weight gains is tested.

Test 2: Test for Statistical Significance of Time Effect on Expected Weight Gain

$$H_0: \delta_t^d = 0 \quad t = 1, 2$$

$$H_1: \delta_t^d \neq 0 \quad t = 1 \text{ or } 2$$

Test 3: Test for Statistical Significance of Time Effect on Variance of Weight Gain

$$H_0: \gamma_t^s = 0 \quad t = 1, 2$$

$$H_1: \gamma_t^s \neq 0 \quad t = 1 \text{ or } 2$$

A failure to reject the null hypothesis in test 1 would imply that the variability of weight gains does not depend on the specified exogenous variables (Griffiths and Anderson, 1982). The null hypothesis in test 2 should be rejected if time effects do not significantly affect the mean. Similarly, the null hypothesis in test 3 should be rejected if time effects do not significantly affect the variance of weight gains. All of the tests are carried out by using a Wald test (Judge et al., 1988).

Results and Discussion

The parameter estimates of the stochastic production function are reported in Table 1. The signs of the estimated parameters of the deterministic term conform with the proposed hypotheses (positive and diminishing marginal product expectations over the relevant range of the sample data). The coefficient of the interaction term is negative, indicating a trade-off between level of forage availability and energy supplements. In

addition, the model allows a good prediction of the observed weight gains; the squared correlation coefficient between the predicted and the observed weight gains is 0.84.

Table 2 summarizes the results of the specification tests. The null hypothesis of all three tests are rejected at the 2.5% level. These results imply that the variance of the error term is heteroskedastic and depends upon the levels of the specified exogenous variables. In addition, time effects significantly affect the expected weight gain and the variance of weight gains.

Following Just and Pope (1978), decreasing, increasing, or constant marginal risk for energy supplements can be determined based on the sign of the first derivative of the stochastic term with respect to energy supplements (h_{EN}). That is, changes in the variability of stocker cattle weight gains are given by:

$$h_{EN} = \frac{\partial V(GN)}{\partial EN} = \frac{\partial h(D, INWT, EN, PF, \alpha)}{\partial EN} \quad [6]$$

where GN denotes weight gain. A positive (negative) sign of h_{EN} implies increasing (decreasing) variability of weight gains with increased use of energy supplement. Given the functional form used for the stochastic component, the sign of h_{EN} can be determined without ambiguity based on the results of the estimated variance equation:

$$V(GN) = \exp(7.428 - 0.020INWT - 0.394EN - 0.027PF - 0.065D1 + 0.399D2) \quad [7]$$

Partially differentiating equation [7] with respect to the energy supplement variable (EN) yields:

$$h_{EN} = -0.394 \exp(7.428 - 0.020INWT - 0.394EN - 0.027PF - 0.065D1 + 0.399D2) \quad [8]$$

Equation [8] implies that the variability of weight gains decreases with increased use of energy supplement, over all energy supplement levels. Thus, energy supplement is a risk-reducing input.

These results are consistent with the actual cattle performance data of this study in which mean \pm standard deviation of weight gains/steer for control, high-fiber and high-starch supplemented cattle were: 237 \pm 39.4 lb (control); 275 \pm 35.9 lb (high-fiber) and 264 \pm 33.8 lb (high-starch). Both types of energy supplements increased mean weight gains and decreased the standard deviations. Results also indicate that as more forage is available less variability of weight gains is observed. That is $\partial V(GN)/\partial PF < 0$, implying that production risk is decreased as forage availability increases.

This risk-reducing character of energy supplement inputs implies that risk-averse producers concerned with reducing income variability can use energy supplementation as a means of reducing production risk. As a result

of reducing weight gain variability, producers can make more reliable projections of cattle weights off wheat pasture. By increasing the certainty of cattle weights off wheat pasture, break-even selling prices can be more accurately calculated. This, in turn, will greatly aid marketing, forward contracting and retained ownership decisions.

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Table 1. Parameter estimates of the stochastic production function, time series and cross-section data over three grazing seasons (1989-90, 1990-91, 1991-92)^a.

Dependent Variable	Independent Variable	Coefficient	Standard Error
Log of Weight Gains	Constant	-6.291**	1.095
	ln (EN)	.674**	.390
	ln (PF)	.017	.042
	ln (INWT)	1.085**	.181
	ln(EN)ln(PF)	-.248**	.125
	D1	-.075**	.028
	D2	-.091**	.029
Log of Variance of Weight Gains	Constant	7.428**	4.657
	INWT	-.020**	.009
	EN	-.394*	.693
	PF	-.027*	.046
	D1	-.065	.602
	D2	.399	.630
R-Square Adjusted		.84	
Number of Observations		45	

^a D1 and D2 denote year dummy variables for 1989-90, 1990-91, respectively. Single and double asterisks denote significance at the 10% and 5% levels, respectively.

Table 2. Specification Test Results^a.

Null hypothesis	Test-statistic	Critical value at the 2.5% level
Variance does not depend on input levels (Test 1)	28.026	9.348
No time effect on mean (Test 2)	46.872	7.378
No time effect on variance (Test 3)	8.690	7.378

^a Under the null hypothesis, the Wald statistic is distributed chi-square with three degrees of freedom for test 1, and two degrees of freedom for Tests 2 and 3.